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Overview The existence of this book is owed (both figuratively and literally) to the fact that the building blocks of matter possess a quality called *charge*. Two important aspects of charge are *conservation* and *quantization*. The electric force between two charges is given by *Coulomb's law*. Like the gravitational force, the electric force falls off like $1/r^2$. It is *conservative*, so we can talk about the potential energy of a system of charges (the work done in assembling them). A very useful concept is the *electric field*, which is defined as the force per unit charge. Every point in space has a unique electric field associated with it. We can define the *flux* of the electric field through a given surface. This leads us to *Gauss's law*, which is an alternative way of stating Coulomb's law. In cases involving sufficient symmetry, it is much quicker to calculate the electric field via Gauss's law than via Coulomb's law and direct integration. Finally, we discuss the *energy density* in the electric field, which provides another way of calculating the potential energy of a system.

Electrostatics: charges and fields

1.1 Electric charge

Electricity appeared to its early investigators as an extraordinary phenomenon. To draw from bodies the “subtle fire,” as it was sometimes called, to bring an object into a highly electrified state, to produce a steady flow of current, called for skillful contrivance. Except for the spectacle of lightning, the ordinary manifestations of nature, from the freezing of water to the growth of a tree, seemed to have no relation to the curious behavior of electrified objects. We know now that electrical

forces largely determine the physical and chemical properties of matter over the whole range from atom to living cell. For this understanding we have to thank the scientists of the nineteenth century, Ampère, Faraday, Maxwell, and many others, who discovered the nature of electromagnetism, as well as the physicists and chemists of the twentieth century who unraveled the atomic structure of matter.

Classical electromagnetism deals with electric charges and currents and their interactions as if all the quantities involved could be measured independently, with unlimited precision. Here *classical* means simply “nonquantum.” The quantum law with its constant h is ignored in the classical theory of electromagnetism, just as it is in ordinary mechanics. Indeed, the classical theory was brought very nearly to its present state of completion before Planck’s discovery of quantum effects in 1900. It has survived remarkably well. Neither the revolution of quantum physics nor the development of special relativity dimmed the luster of the electromagnetic field equations Maxwell wrote down 150 years ago.

Of course the theory was solidly based on experiment, and because of that was fairly secure within its original range of application – to coils, capacitors, oscillating currents, and eventually radio waves and light waves. But even so great a success does not guarantee validity in another domain, for instance, the inside of a molecule.

Two facts help to explain the continuing importance in modern physics of the classical description of electromagnetism. First, special relativity required no revision of classical electromagnetism. Historically speaking, special relativity *grew out* of classical electromagnetic theory and experiments inspired by it. Maxwell’s field equations, developed long before the work of Lorentz and Einstein, proved to be entirely compatible with relativity. Second, quantum modifications of the electromagnetic forces have turned out to be unimportant down to distances less than 10^{-12} meters, 100 times smaller than the atom. We can describe the repulsion and attraction of particles in the atom using the same laws that apply to the leaves of an electroscope, although we need quantum mechanics to predict how the particles will behave under those forces. For still smaller distances, a fusion of electromagnetic theory and quantum theory, called *quantum electrodynamics*, has been remarkably successful. Its predictions are confirmed by experiment down to the smallest distances yet explored.

It is assumed that the reader has some acquaintance with the elementary facts of electricity. We are not going to review all the experiments by which the existence of electric charge was demonstrated, nor shall we review all the evidence for the electrical constitution of matter. On the other hand, we do want to look carefully at the experimental foundations of the basic laws on which all else depends. In this chapter we shall study the physics of stationary electric charges – *electrostatics*.

Certainly one fundamental property of electric charge is its existence in the two varieties that were long ago named *positive* and *negative*.

The observed fact is that all charged particles can be divided into two classes such that all members of one class repel each other, while attracting members of the other class. If two small electrically charged bodies *A* and *B*, some distance apart, attract one another, and if *A* attracts some third electrified body *C*, then we always find that *B* repels *C*. Contrast this with gravitation: there is only one kind of gravitational mass, and every mass attracts every other mass.

One may regard the two kinds of charge, positive and negative, as opposite manifestations of one quality, much as *right* and *left* are the two kinds of handedness. Indeed, in the physics of elementary particles, questions involving the sign of the charge are sometimes linked to a question of handedness, and to another basic symmetry, the relation of a sequence of events, *a*, then *b*, then *c*, to the temporally reversed sequence *c*, then *b*, then *a*. It is only the duality of electric charge that concerns us here. For every kind of particle in nature, as far as we know, there can exist an *antiparticle*, a sort of electrical “mirror image.” The antiparticle carries charge of the opposite sign. If any other intrinsic quality of the particle has an opposite, the antiparticle has that too, whereas in a property that admits no opposite, such as mass, the antiparticle and particle are exactly alike.

The electron’s charge is negative; its antiparticle, called a *positron*, has a positive charge, but its mass is precisely the same as that of the electron. The proton’s antiparticle is called simply an *antiproton*; its electric charge is negative. An electron and a proton combine to make an ordinary hydrogen atom. A positron and an antiproton could combine in the same way to make an atom of antihydrogen. Given the building blocks, positrons, antiprotons, and antineutrons,¹ there could be built up the whole range of antimatter, from antihydrogen to antigalaxies. There is a practical difficulty, of course. Should a positron meet an electron or an antiproton meet a proton, that pair of particles will quickly vanish in a burst of radiation. It is therefore not surprising that even positrons and antiprotons, not to speak of antiatoms, are exceedingly rare and short-lived in our world. Perhaps the universe contains, somewhere, a vast concentration of antimatter. If so, its whereabouts is a cosmological mystery.

The universe around us consists overwhelmingly of matter, not antimatter. That is to say, the abundant carriers of negative charge are electrons, and the abundant carriers of positive charge are protons. The proton is nearly 2000 times heavier than the electron, and very different, too, in some other respects. Thus matter at the atomic level incorporates negative and positive electricity in quite different ways. The positive charge is all in the atomic nucleus, bound within a massive structure no more than 10^{-14} m in size, while the negative charge is spread, in

¹ Although the electric charge of each is zero, the neutron and its antiparticle are not interchangeable. In certain properties that do not concern us here, they are opposite.

effect, through a region about 10^4 times larger in dimensions. It is hard to imagine what atoms and molecules – and all of chemistry – would be like, if not for this fundamental electrical asymmetry of matter.

What we call negative charge, by the way, could just as well have been called positive. The name was a historical accident. There is nothing essentially negative about the charge of an electron. It is not like a negative integer. A negative integer, once multiplication has been defined, differs essentially from a positive integer in that its square is an integer of opposite sign. But the product of two charges is not a charge; there is no comparison.

Two other properties of electric charge are essential in the electrical structure of matter: charge is *conserved*, and charge is *quantized*. These properties involve *quantity* of charge and thus imply a measurement of charge. Presently we shall state precisely how charge can be measured in terms of the force between charges a certain distance apart, and so on. But let us take this for granted for the time being, so that we may talk freely about these fundamental facts.

1.2 Conservation of charge

The total charge in an isolated system never changes. By *isolated* we mean that no matter is allowed to cross the boundary of the system. We could let light pass into or out of the system, since the “particles” of light, called *photons*, carry no charge at all. Within the system charged particles may vanish or reappear, but they always do so in pairs of equal and opposite charge. For instance, a thin-walled box in a vacuum exposed to gamma rays might become the scene of a “pair-creation” event in which a high-energy photon ends its existence with the creation of an electron and a positron (Fig. 1.1). Two electrically charged particles have been newly created, but the net change in total charge, in and on the box, is zero. An event that *would* violate the law we have just stated would be the creation of a positively charged particle *without* the simultaneous creation of a negatively charged particle. Such an occurrence has never been observed.

Of course, if the electric charges of an electron and a positron were not precisely equal in magnitude, pair creation would still violate the strict law of charge conservation. That equality is a manifestation of the particle–antiparticle duality already mentioned, a universal symmetry of nature.

One thing will become clear in the course of our study of electromagnetism: nonconservation of charge would be quite incompatible with the structure of our present electromagnetic theory. We may therefore state, either as a postulate of the theory or as an empirical law supported without exception by all observations so far, the charge conservation law:

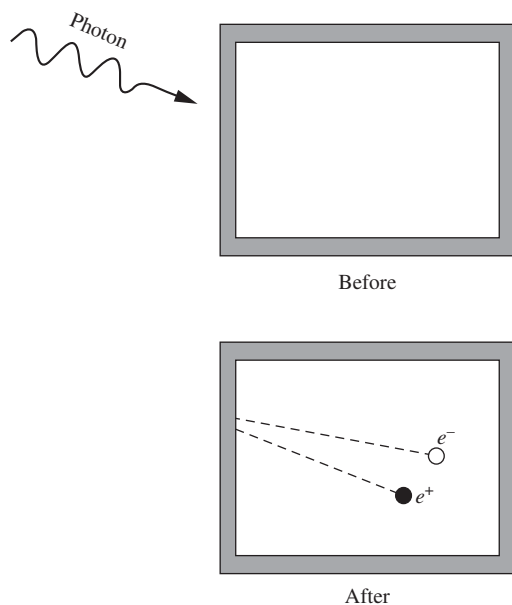


Figure 1.1.
Charged particles are created in pairs with equal and opposite charge.

The total electric charge in an isolated system, that is, the algebraic sum of the positive and negative charge present at any time, never changes.

Sooner or later we must ask whether this law meets the test of relativistic invariance. We shall postpone until Chapter 5 a thorough discussion of this important question. But the answer is that it does, and not merely in the sense that the statement above holds in any given inertial frame, but in the stronger sense that observers in different frames, measuring the charge, obtain the same number. In other words, the total electric charge of an isolated system is a relativistically invariant number.

1.3 Quantization of charge

The electric charges we find in nature come in units of one magnitude only, equal to the amount of charge carried by a single electron. We denote the magnitude of that charge by e . (When we are paying attention to sign, we write $-e$ for the charge on the electron itself.) We have already noted that the positron carries precisely that amount of charge, as it must if charge is to be conserved when an electron and a positron annihilate, leaving nothing but light. What seems more remarkable is the apparently exact equality of the charges carried by all other charged particles – the equality, for instance, of the positive charge on the proton and the negative charge on the electron.

That particular equality is easy to test experimentally. We can see whether the net electric charge carried by a hydrogen molecule, which consists of two protons and two electrons, is zero. In an experiment carried out by J. G. King,² hydrogen gas was compressed into a tank that was electrically insulated from its surroundings. The tank contained about $5 \cdot 10^{24}$ molecules (approximately 17 grams) of hydrogen. The gas was then allowed to escape by means that prevented the escape of any ion – a molecule with an electron missing or an extra electron attached. If the charge on the proton differed from that on the electron by, say, one part in a billion, then each hydrogen molecule would carry a charge of $2 \cdot 10^{-9}e$, and the departure of the whole mass of hydrogen would alter the charge of the tank by $10^{16}e$, a gigantic effect. In fact, the experiment could have revealed a residual molecular charge as small as $2 \cdot 10^{-20}e$, and none was observed. This proved that the proton and the electron do not differ in magnitude of charge by more than 1 part in 10^{20} .

Perhaps the equality is really *exact* for some reason we don't yet understand. It may be connected with the possibility, suggested by certain

² See King (1960). References to previous tests of charge equality will be found in this article and in the chapter by V. W. Hughes in Hughes (1964).

theories, that a proton can, *very* rarely, decay into a positron and some uncharged particles. If that were to occur, even the slightest discrepancy between proton charge and positron charge would violate charge conservation. Several experiments designed to detect the decay of a proton have not yet, as of this writing, registered with certainty a single decay. If and when such an event is observed, it will show that exact equality of the magnitude of the charge of the proton and the charge of the electron (the positron's antiparticle) can be regarded as a corollary of the more general law of charge conservation.

That notwithstanding, we now know that the *internal* structure of all the strongly interacting particles called *hadrons* – a class that includes the proton and the neutron – involves basic units called *quarks*, whose electric charges come in multiples of $e/3$. The proton, for example, is made with three quarks, two with charge $2e/3$ and one with charge $-e/3$. The neutron contains one quark with charge $2e/3$ and two quarks with charge $-e/3$.

Several experimenters have searched for single quarks, either free or attached to ordinary matter. The fractional charge of such a quark, since it cannot be neutralized by any number of electrons or protons, should betray the quark's presence. So far no fractionally charged particle has been conclusively identified. The present theory of the strong interactions, called *quantum chromodynamics*, explains why the liberation of a quark from a hadron is most likely impossible.

The fact of charge quantization lies outside the scope of classical electromagnetism, of course. We shall usually ignore it and act as if our point charges q could have any strength whatsoever. This will not get us into trouble. Still, it is worth remembering that classical theory cannot be expected to explain the structure of the elementary particles. (It is not certain that present quantum theory can either!) What holds the electron together is as mysterious as what fixes the precise value of its charge. Something more than electrical forces must be involved, for the electrostatic forces between different parts of the electron would be repulsive.

In our study of electricity and magnetism we shall treat the charged particles simply as carriers of charge, with dimensions so small that their extension and structure is, for most purposes, quite insignificant. In the case of the proton, for example, we know from high-energy scattering experiments that the electric charge does not extend appreciably beyond a radius of 10^{-15} m. We recall that Rutherford's analysis of the scattering of alpha particles showed that even heavy nuclei have their electric charge distributed over a region smaller than 10^{-13} m. For the physicist of the nineteenth century a "point charge" remained an abstract notion. Today we are on familiar terms with the atomic particles. The graininess of electricity is so conspicuous in our modern description of nature that we find a point charge less of an artificial idealization than a smoothly varying distribution of charge density. When we postulate such smooth charge distributions, we may think of them as averages over very

large numbers of elementary charges, in the same way that we can define the macroscopic density of a liquid, its lumpiness on a molecular scale notwithstanding.

1.4 Coulomb's law

As you probably already know, the interaction between electric charges at rest is described by Coulomb's law: two stationary electric charges repel or attract one another with a force proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance between them.

We can state this compactly in vector form:

$$\mathbf{F}_2 = k \frac{q_1 q_2 \hat{\mathbf{r}}_{21}}{r_{21}^2}. \quad (1.1)$$

Here q_1 and q_2 are numbers (scalars) giving the magnitude and sign of the respective charges, $\hat{\mathbf{r}}_{21}$ is the unit vector in the direction³ from charge 1 to charge 2, and $\mathbf{F}_2$ is the force acting on charge 2. Thus Eq. (1.1) expresses, among other things, the fact that like charges repel and unlike charges attract. Also, the force obeys Newton's third law; that is, $\mathbf{F}_2 = -\mathbf{F}_1$.

The unit vector $\hat{\mathbf{r}}_{21}$ shows that the force is parallel to the line joining the charges. It could not be otherwise unless space itself has some built-in directional property, for with two point charges alone in empty and isotropic space, no other direction could be singled out.

If the point charge itself had some internal structure, with an axis defining a direction, then it would have to be described by more than the mere scalar quantity q . It is true that some elementary particles, including the electron, do have another property, called *spin*. This gives rise to a magnetic force between two electrons in addition to their electrostatic repulsion. This magnetic force does not, in general, act in the direction of the line joining the two particles. It decreases with the inverse fourth power of the distance, and at atomic distances of 10^{-10} m the Coulomb force is already about 10^4 times stronger than the magnetic interaction of the spins. Another magnetic force appears if our charges are moving – hence the restriction to stationary charges in our statement of Coulomb's law. We shall return to these magnetic phenomena in later chapters.

Of course we must assume, in writing Eq. (1.1), that both charges are well localized, each occupying a region small compared with r_{21} . Otherwise we could not even define the distance r_{21} precisely.

The value of the constant k in Eq. (1.1) depends on the units in which r , $\mathbf{F}$, and q are to be expressed. In this book we will use the International System of Units, or “SI” units for short. This system is based on the

³ The convention we adopt here may not seem the natural choice, but it is more consistent with the usage in some other parts of physics and we shall try to follow it throughout this book.